Chp-6:Lecture Goals

- Serviceability
- Deflection calculation
- Deflection example

- Structural Design Profession is concerned with:

  Limit States Philosophy:

Strength Limit State

(safety-fracture, fatigue, overturning buckling etc.-)

Serviceability

Performance of structures under normal service load and are concerned the uses and/or occupancy of structures

The magnitudes of deflections for concrete members can be quite important.

Excessive deflections of beams and slabs may cause ;

sagging floors,

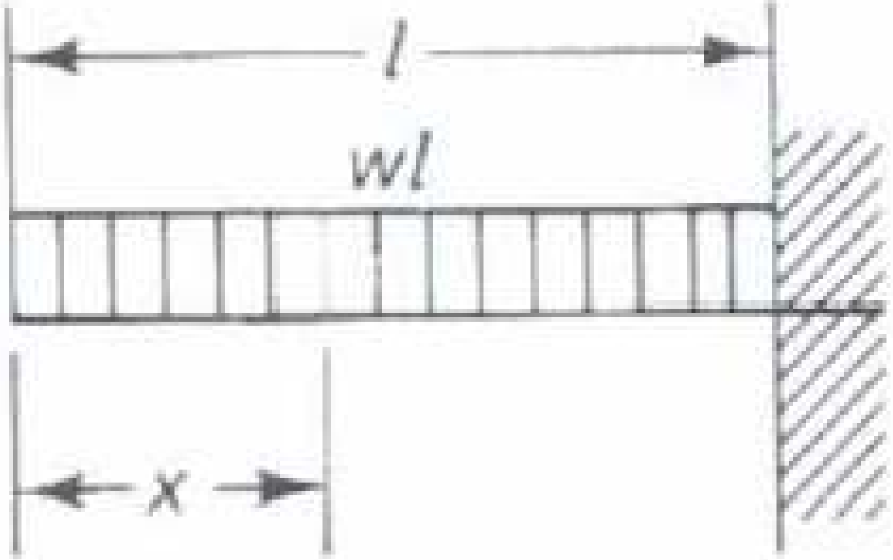
ponding on flat roofs,

excessive vibrations,

interference with the proper operation of supported machinery.

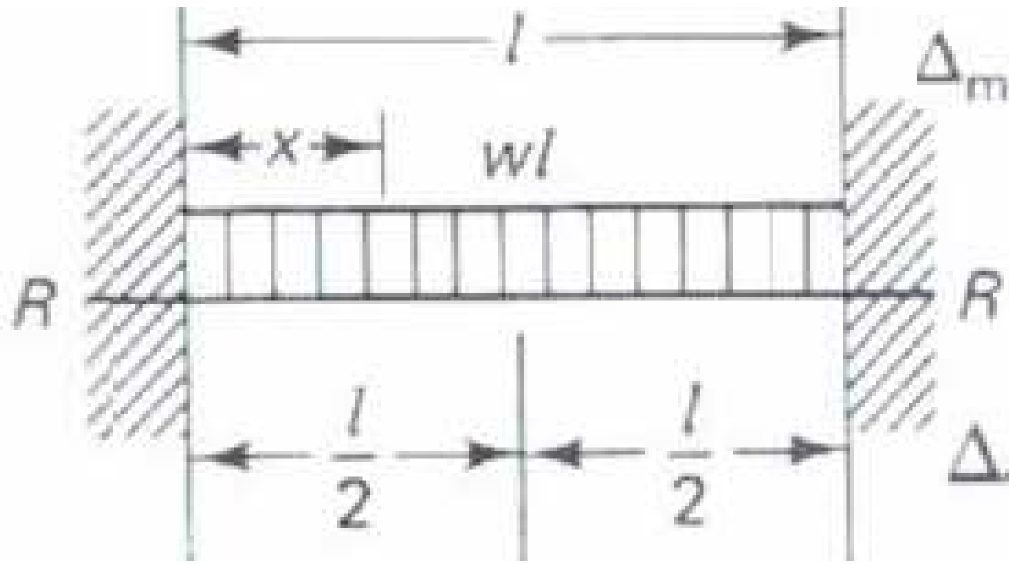
damage partitions and cause poor fitting of doors and windows.

damage structure's appearance or frighten the occupants



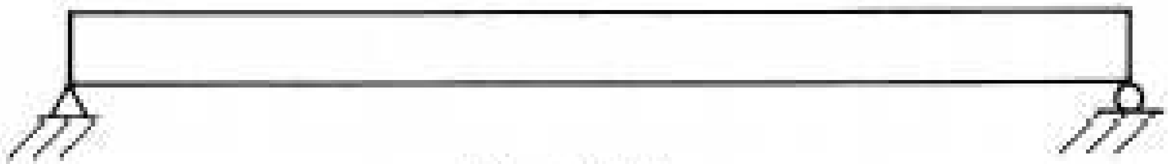
$$\Delta_{max} \text{ (at free end)} = \frac{wl^4}{8EI}$$

$$\Delta x = \frac{w}{24EI}(x^4 - 4l^3x + 3l^4)$$

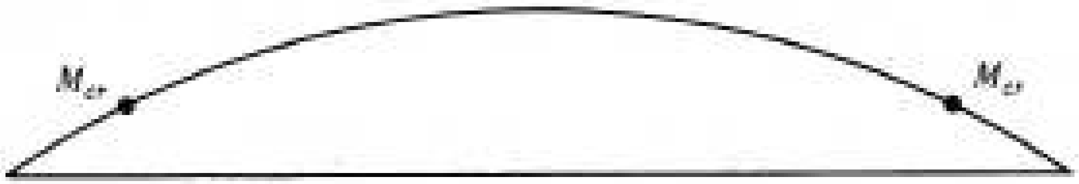


$$\Delta_{max} \text{ (at center)} = \frac{wl^4}{384EI}$$

$$\Delta x = \frac{wx^2}{24EI}(l - x)^2$$

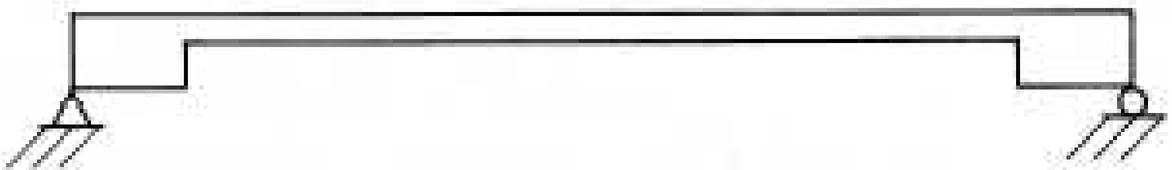


(a) Actual beam



(b) Moment diagram

(c) Cracks where $M \geq M_{cr}$



(d) Effect of cracks on effective beam cross section

$$\lambda_{\Delta} = \frac{\xi}{1+50\rho'} \qquad \delta_L = \delta_{D+L} - \delta_D \qquad \delta_{LT} = \delta_L + \lambda_{\infty}\delta_D + \lambda_t \delta_{SL}$$

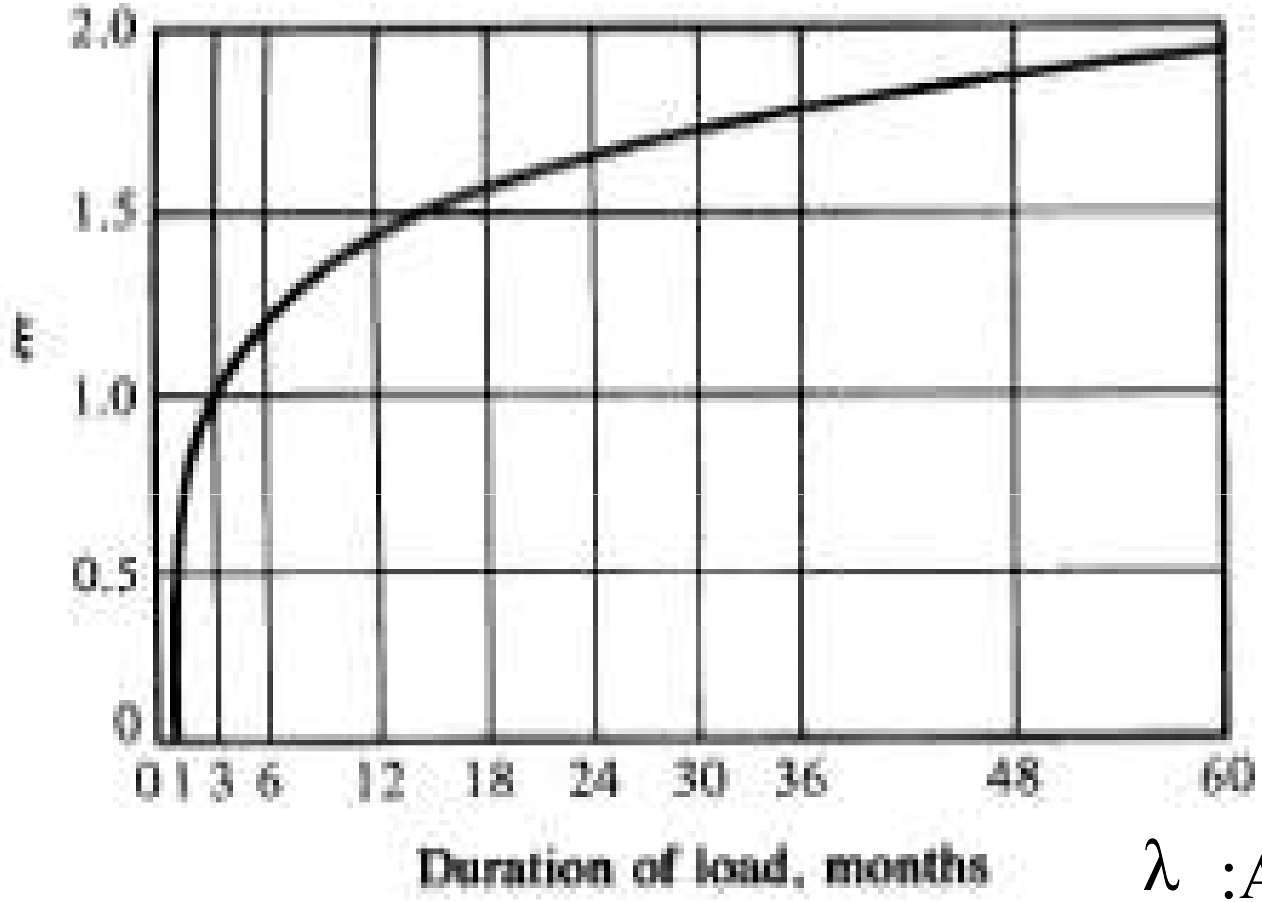


Figure 6.4 Multipliers for long-time deflections. (ACI Commentary Figure R9.5.2.5.)

λ :Amplification factor

ζ :Time dependent Factor

The steps involved in calculating instantaneous and long-term deflections can be summarized as follows:

(a) Compute the instantaneous or short-term deflection δ_D for dead load only.

(b) Compute instantaneous deflection δ_{D+L} for dead plus full live load.

(c) Determine instantaneous deflection δ_L for full live load only.

(d) Compute instantaneous deflection due to dead load plus the sustained part of the live load $\delta_D + \delta_{SL}$.

(e) Determine instantaneous deflection δ_L for the part of the live load that is sustained.

(f) Determine the long-term deflection for dead load plus the sustained part of the live load δ_{LT}.

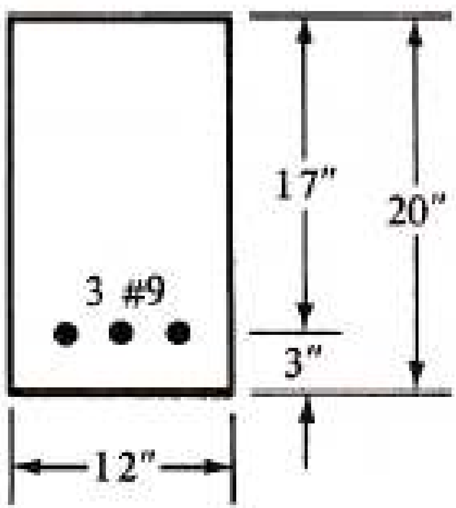
the deflections calculated as described

exceed certain limits, depending on the type of structure. Maximum deflections permitted by the ACI for several floor and roof situations were presented in Table 6.1.

Table 6.1 Maximum Permissible Computed Deflections

Type of member	Deflection to be considered	Deflection limitation
Flat roofs not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load L	$\frac{\ell^{*}}{180}$
Floors not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load L	$\frac{\ell}{360}$
Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections	That part of the total deflection occurring after attachment of nonstructural elements (sum of the long-term deflection due to all sustained loads and the immediate deflection due to any additional live load)[†]	$\frac{\ell^{‡}}{480}$
Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections		$\frac{\ell^{§}}{240}$

EXAMPLE 6.1 The beam has a simple span of 20 ft and supports a dead load including its own weight of 1 klf and a live load of 0.7 klf. $f'_c = 3000$ psi.



(a) Calculate the instantaneous deflection for $D + L$.

(b) Calculate the deflection assuming that 30% of the live load is continuously applied for three years.

Solution: **(a) Instantaneous or short-term dead load deflection (δ_D)**

$$I_g = \left(\tfrac{1}{12}\right)(12)(20)^3 = 8000 \text{ in.}^4$$

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{(7.5\sqrt{3000})(8000)}{10} = 328{,}633 \text{ in.-lb} = 27.4 \text{ ft-k}$$

$$M_a = \frac{(1.0)(20)^2}{8} = 50 \text{ ft-k} = M_D$$

By transformed-area calculations the values of x and I_{cr} can be determined

$x = 6.78''$

See Example 2.2

$I_{cr} = 4067 \text{ in.}^4$

$$I_{e(\text{midspan})} = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left(1 - \left(\frac{M_{cr}}{M_a}\right)^3\right) I_{cr}$$

Ma: Max. Service Load Moment

$$I_e = \left(\frac{27.4}{50}\right)^3 (8000) + \left[1 - \left(\frac{27.4}{50}\right)^3\right] 4067 = 4714 \text{ in.}^4$$

$$E_c = 57{,}000\sqrt{3000} = 3.122 \times 10^6 \text{ psi}$$

$$\delta_D = \frac{5wl^4}{384E_cI_e} = \frac{(5)\left(\frac{1000}{12}\right)(12 \times 20)^4}{(384)(3.122 \times 10^6)(4714)} = \underline{\underline{0.245 \text{ in.}^*}}$$

(b) Instantaneous or short-term deflection for dead + full live load (δ_{D+L})

$$M_a = \frac{(1.7)(20)^2}{8} = 85 \text{ ft-k}$$

Noting that the value of I_e changes when the moments change

$$I_e = \left(\frac{27.4}{85}\right)^3 (8000) + \left[1 - \left(\frac{27.4}{85}\right)^3\right](4067) = 4199 \text{ in.}^4$$

$$\delta_{D+L} = \frac{(5)\left(\frac{1700}{12}\right)(12 \times 20)^4}{(384)(3.122 \times 10^6)(4199)} = 0.467 \text{ in.*}$$

(c) Initial deflection for full live load (δ_L)

$$\delta_L = \delta_{D+L} - \delta_D = 0.467 - 0.245 = \underline{\underline{0.222 \text{ in.*}}}$$

This is the live load deflection that would be compared with the first or second row of Table 6.1. If the beam is part of a floor system that is "not supporting or attached to nonstructural elements likely to be damaged by large deflections" (left column of Table 6.1), then the deflection limit is $\ell/360 = (20)(12)/360 = 0.67$ in. This limit would easily be satisfied in this case as the calculated deflection is only 0.22 in.

EXAMPLE 6.2

Determine the instantaneous deflection at the midspan of the continuous T beam shown in Figure 6.7(a). The member supports a dead load including its own weight of 1.5 k/ft and a live load of 2.5 k/ft. $f_c' = 3000$ psi and $n = 9$. The moment diagram for full dead and live loads is shown in Figure 6.7(b), and the beam cross section is shown in Figure 6.7(c).

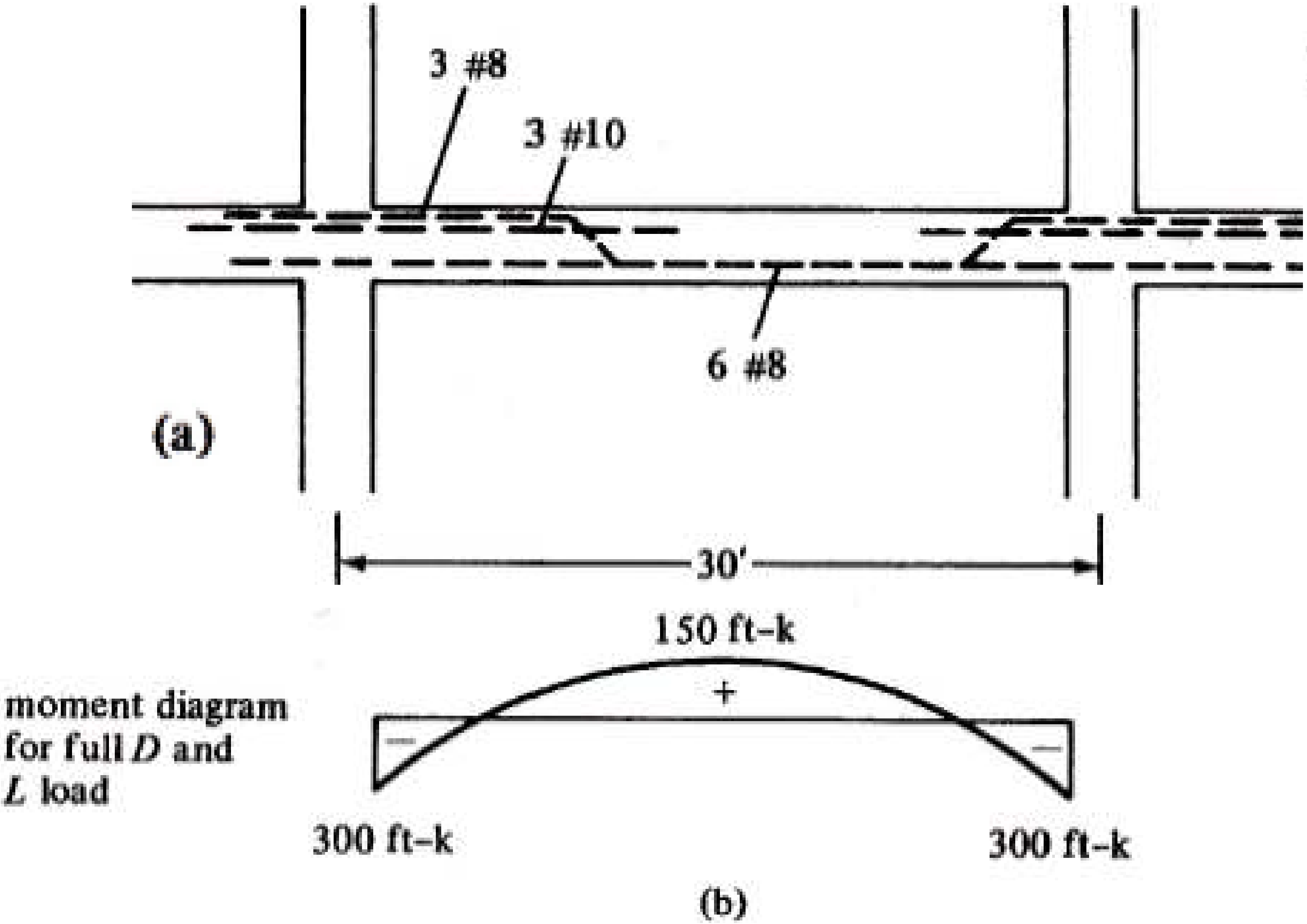
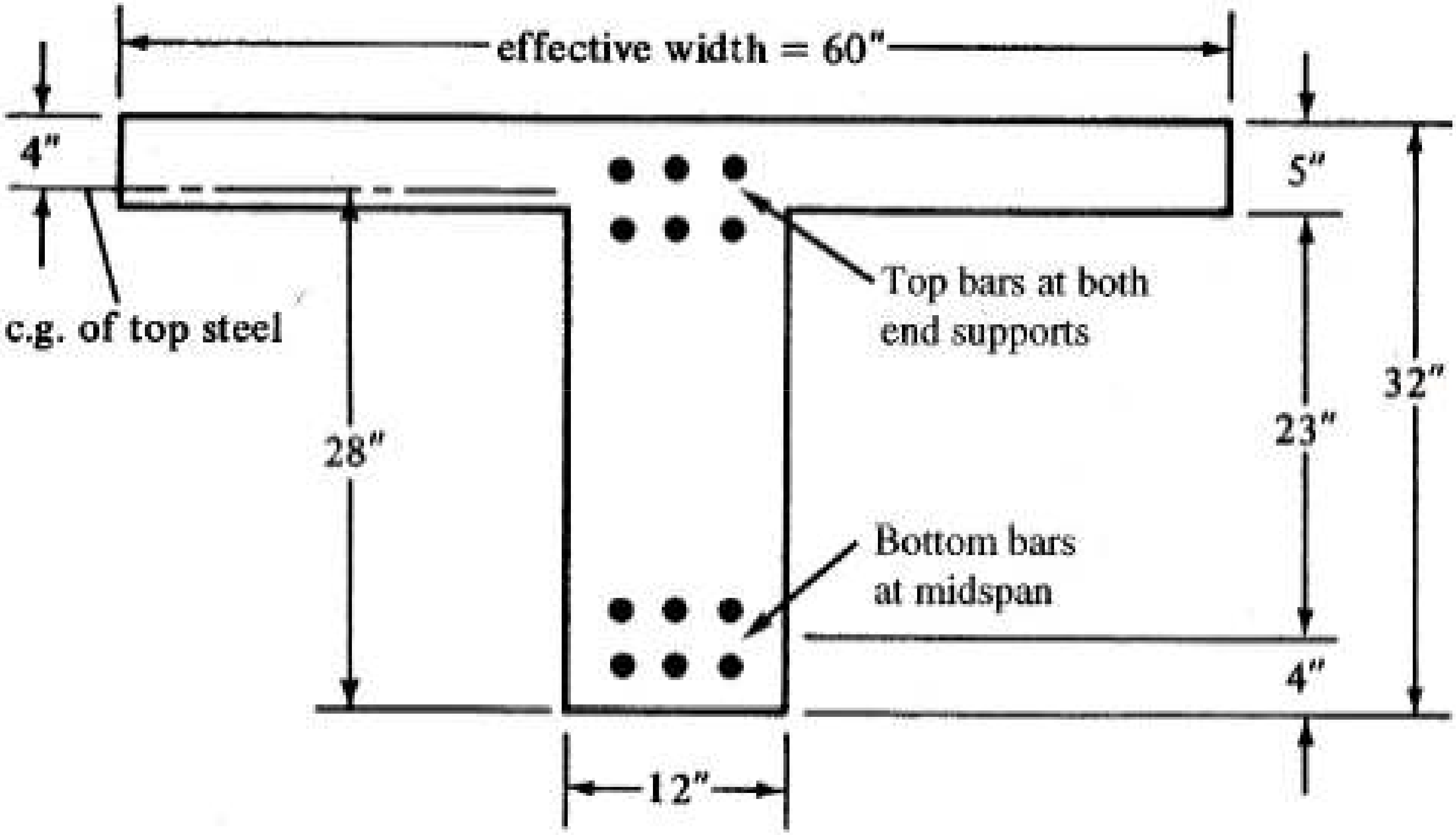


Figure 6.7

Figure 6.7


effective width = 60"
4"
c.g. of top steel
5"
Top bars at both end supports
28"
23"
32"
Bottom bars at midspan
4"
12"

(c)

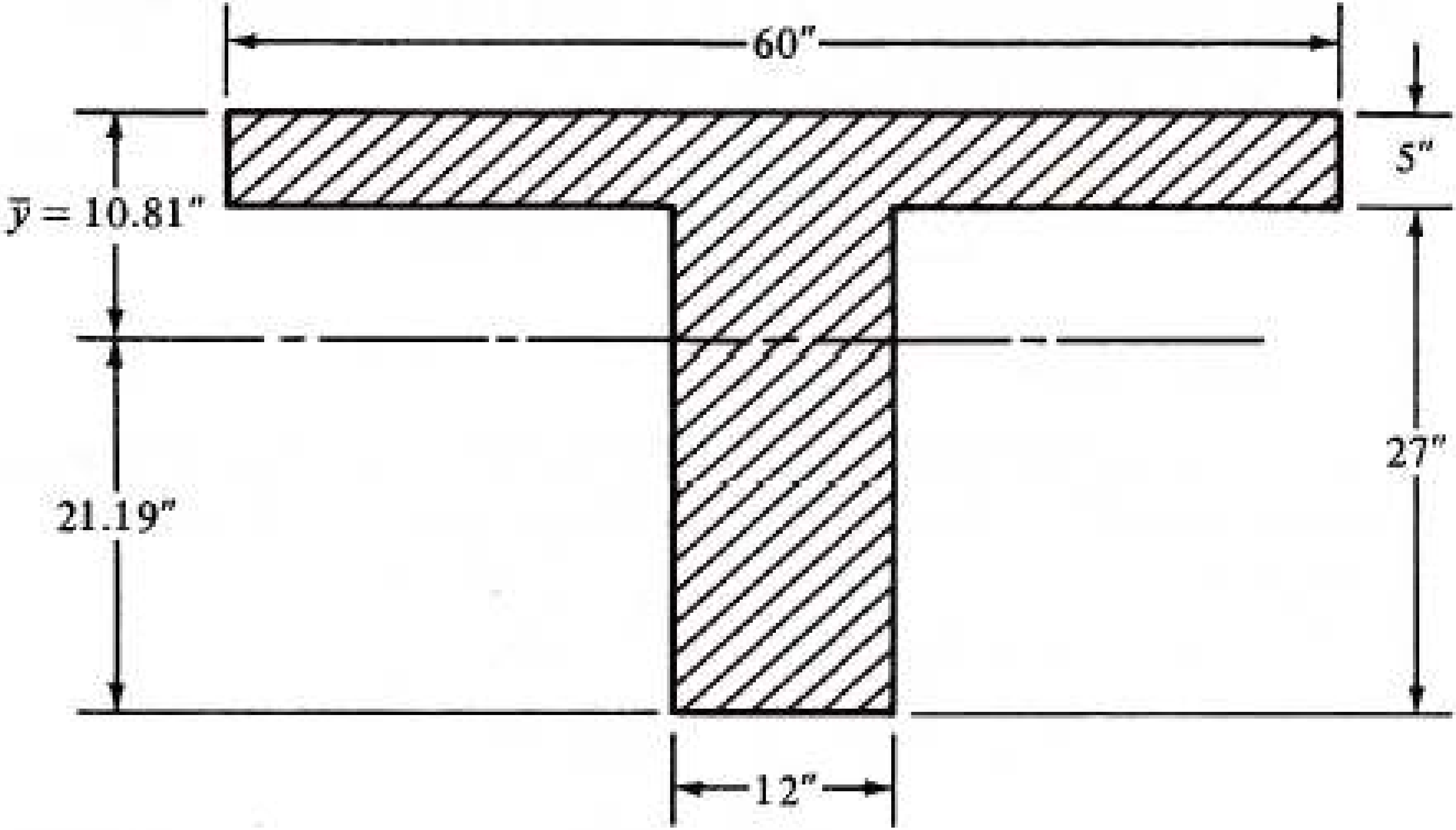


Figure 6.8

For Positive-Moment Region

1. Locating centroidal axis for uncracked section and calculating gross moment of inertia I_g and cracking moment M_{cr} for the positive-moment region (Figure 6.8)

$$\bar{y} - 10.81''$$

$$I_g = 60{,}185 \text{ in.}^4$$

$$M_{cr} = \frac{(7.5)(\sqrt{3000})(60{,}185)}{21.19} = 1{,}166{,}754 \text{ in.-lb} = 97.2 \text{ ft-k}$$

2. Locating centroidal axis of cracked section and calculating transformed moment of inertia I_{cr} for the positive-moment region (Figure 6.9)

$$x = 5.65''$$

$$I_{cr} = 24{,}778 \text{ in.}^4$$

3. Calculating the effective moment of inertia in the positive-moment region

$$M_a = 150 \text{ ft-k}$$

$$I_e = \left(\frac{97.2}{150}\right)^3 (60{,}185) + \left[1 - \left(\frac{97.2}{150}\right)^3\right] 24{,}778 = 34{,}412 \text{ in.}^4$$

For Negative-Moment Region

1. Locating the centroidal axis for uncracked section and calculating gross moment of inertia I_g and cracking moment M_{cr} for the negative-moment region, considering only the hatched rectangle shown in Figure 6.10

$$\bar{y} = \left(\frac{32}{2}\right) = 16''$$

$$I_g = \left(\frac{1}{12}\right)(12)(32)^3 = 32{,}768 \text{ in.}^4$$

$$M_{cr} = \frac{(7.5)(\sqrt{3000})(32{,}768)}{16} = 841{,}302 \text{ in.-lb} = 70.1 \text{ ft-k}$$

The Code does not require that the designer ignore the flanges in tension for this calculation. The authors used this method to be conservative. If the tension flanges are considered, then the cracking moment is calculated from the section in Figure 6.8. The value of $\bar{y}$ is taken to the top of the section (10.81″) because the top is in tension for negative moment, so

$$M_{cr} = \frac{7.5\sqrt{3000}(60{,}185)}{10.81} = 2{,}287{,}096 \text{ in-lb} = 190.6 \text{ ft-k}$$

If this larger value for M_{cr} were used in step 3 below, the value of I_e would be 33,400 in^4.

2. Locating the centroidal axis of the cracked section and calculating the transformed moment of inertia I_{cr} for the negative-moment region (Figure 6.11). See Example 2.5 for this type of calculation.

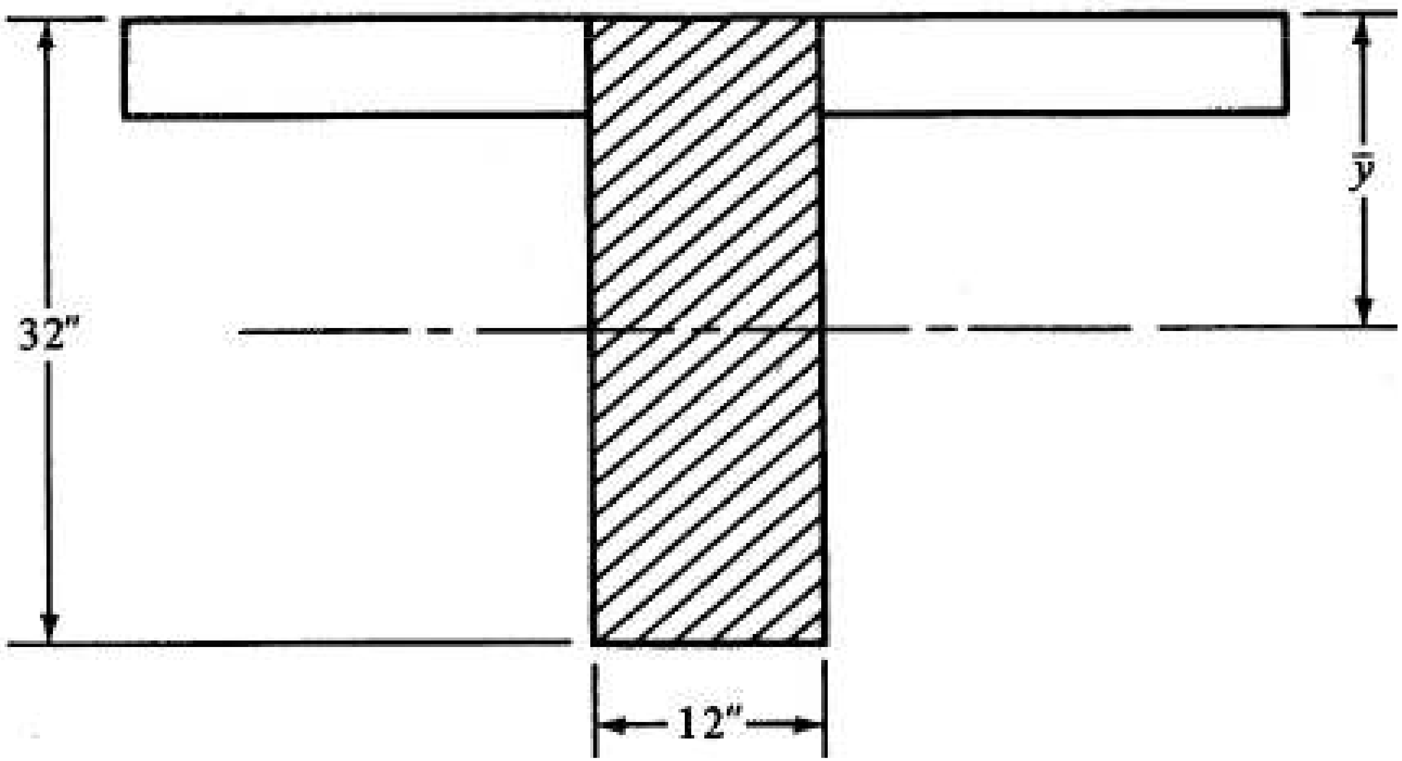
$$x = 10.43''$$

$$I_{cr} = 24{,}147 \text{ in.}^4$$

3. Calculating the effective moment of inertia in the negative-moment region

$$M_a = 300 \text{ ft-k}$$

$$I_e = \left(\frac{70.1}{300}\right)^3 (32{,}768) + \left[1 - \left(\frac{70.1}{300}\right)^3\right] 24{,}147 = 24{,}257 \text{ in.}^4$$



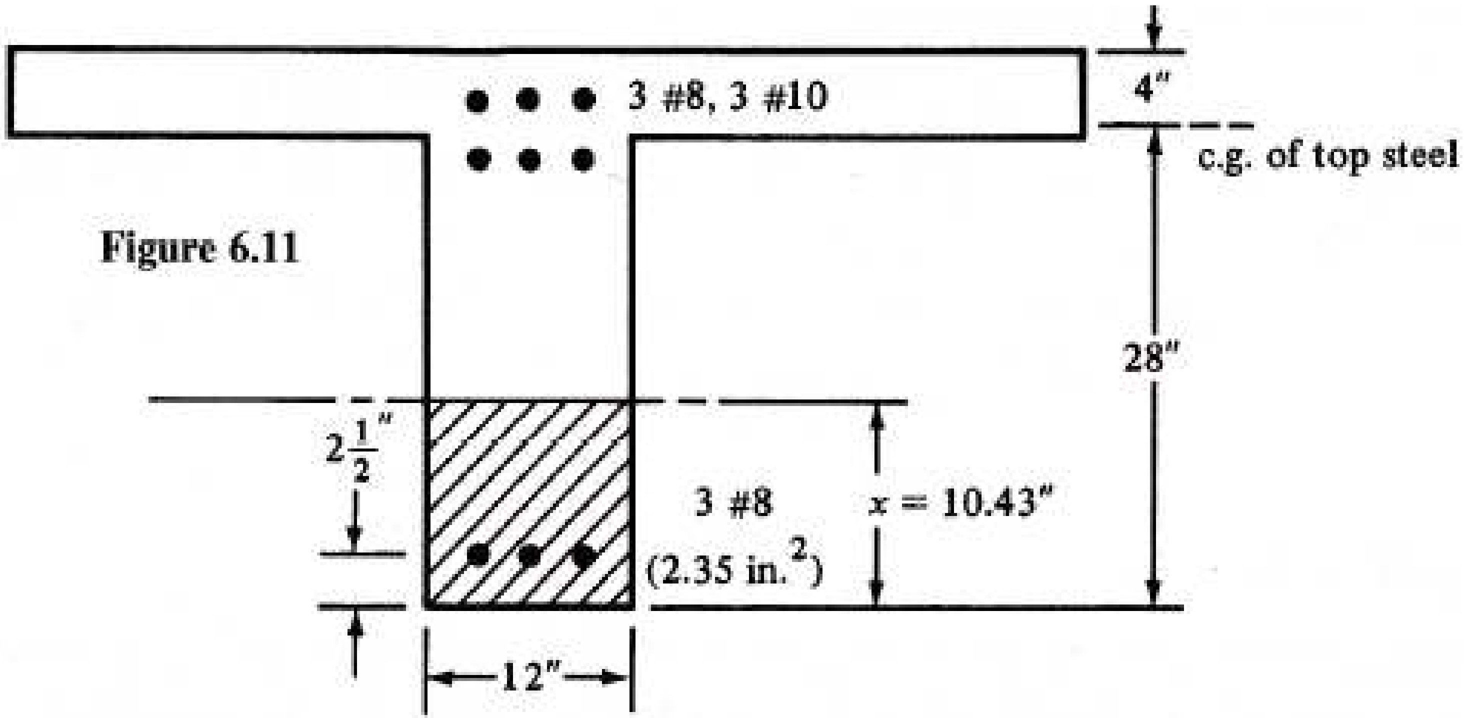


Figure 6.11

Instantaneous Deflection

The I_e to be used is obtained by averaging the I_e at the positive-moment section, with the average of I_e computed at the negative-moment sections at the ends of the span:

$$\text{Average } I_e = \frac{1}{2}\left[\left(\frac{24{,}257 + 24{,}257}{2}\right) + 34{,}412\right] = 29{,}334 \text{ in.}^4$$

$$E_c = 57{,}000\sqrt{3000} = 3.122 \times 10^6 \text{ psi}$$

Using the equation from Figure 6.2(b) and using only live loads to calculate deflections,

$$\delta_L = \frac{w_L l^4}{384 E_c I_e} = \frac{(2.5)(30)^4}{(384)(3122)(29{,}334)}(1728) = 0.10 \text{ in.}$$

In this case the authors used an approximate method to calculate δ_L. Instead of the cumbersome equation ($\delta_L = \delta_{D+L} - \delta_D$) we used earlier in Example 6.1(c), we simply used w_L as the load in the above equation and average I_e. This approximation ignores the difference between I_e for dead load compared with I_e for dead and live load. This method gives a larger deflection, so it is conservative. Many designers have conservative approximations that they try first on many engineering calculations. If they work, there is no need to carry out the more cumbersome ones.

It has been shown that for continuous spans the Code (9.5.2.4) suggests an averaging of the I_e values at the critical positive- and negative-moment sections. The ACI Commentary (R9.5.2.4) says that for approximate deflection calculations for continuous prismatic members it is satisfactory to use the midspan section properties for simple and continuous spans and at supports for cantilevers. This is because these properties, which include the effect of cracking, have the greatest effect on deflections.

Reinforced Concrete Sections - Example

Given a doubly reinforced beam with h = 24 in, b = 12 in., d' = 2.5 in. and d = 21.5 in. with 2# 7 bars in compression steel and 4 # 7 bars in tension steel. The material properties are f_c = 4 ksi and f_y= 60 ksi.

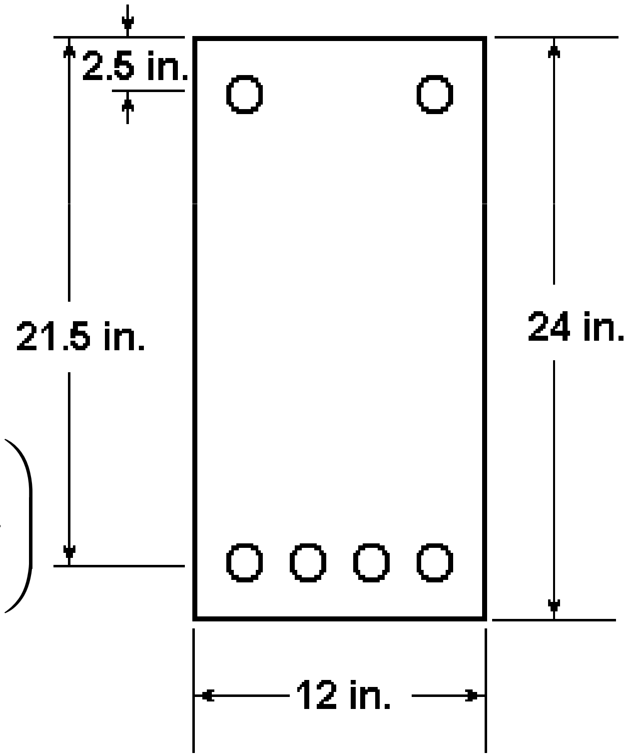
Determine I_{gt}, I_{cr} , $M_{cr(+)}$, $M_{cr(-)}$, and compare to the NA of the beam.

Reinforced Concrete Sections - Example

The components of the beam

$$A_s' = 2\left(0.6 \text{ in}^2\right) = 1.2 \text{ in}^2$$

$$A_s = 4\left(0.6 \text{ in}^2\right) = 2.4 \text{ in}^2$$



$$E_c = 57000\sqrt{f_c'} = 57000\sqrt{4000}\left(\frac{1 \text{ k}}{1000 \text{ lb}}\right)$$

$$= 3605 \text{ ksi}$$

Reinforced Concrete Sections - Example

The compute the n value and the centroid, I uncracked

$$n = \frac{E_s}{E_c} = \frac{29000 \text{ ksi}}{3605 \text{ ksi}} = 8.04$$

	n	area (in^2)	n*area (in^2)	y_i (in)	y_i *n*area (in^2)	I (in^4)	d (in)	d^2*n*area(in^4)
A'_s	7.04	1.2	8.448	2.5	21.12	-	-9.756	804.10
A_s	7.04	2.4	16.896	21.5	363.26	-	9.244	1443.75
A_c	1	288	288	12	3456.00	13824	-0.256	18.89
			313.344		3840.38	13824		2266.74

Reinforced Concrete Sections - Example

The compute the centroid and I uncracked

$$\bar{y} = \frac{\sum y_i n_i A_i}{\sum n_i A_i} = \frac{3840.38 \text{ in}^3}{313.34 \text{ in}^2} = 12.26 \text{ in.}$$

$$\text{I} = \sum I_i + \sum d_i^2 n_i A_i = 13824 \text{ in}^4 + 2266.7 \text{ in}^4$$

$$= 16090.7 \text{ in}^4$$

Reinforced Concrete Sections - Example

The compute the centroid and I for a cracked doubly reinforced beam.

$$\bar{y}^2 + \frac{2(n-1)A_s' + 2nA_s}{b}\bar{y} - \frac{2(n-1)A_s' + 2nA_s d}{b} = 0$$

$$\bar{y}^2 + \frac{2(7.04)(1.2\text{ in}^2) + 2(8.04)(2.4\text{ in}^2)}{12\text{ in.}}\bar{y}$$

$$-\frac{2(7.04)(1.2\text{ in}^2)_s + 2(8.04)(2.4\text{ in}^2)(21.5\text{ in.})}{12\text{ in.}} = 0$$

$$\bar{y}^2 + 4.624\bar{y} - 72.664 = 0$$

Reinforced Concrete Sections - Example

The compute the centroid for a cracked doubly reinforced beam.

$$\bar{y}^2 + 4.624\bar{y} - 72.664 = 0$$

$$\bar{y} = \frac{-4.624 + \sqrt{(4.624)^2 + 4(72.664)}}{2}$$

$$= 6.52 \text{ in.}$$

Reinforced Concrete Sections - Example

The compute the moment of inertia for a cracked doubly reinforced beam.

$$I_{\text{cr}} = \frac{1}{3} b \bar{y}^3 + (n-1) A_s' (\bar{y} - d')^2 + n A_s (d - \bar{y})^2$$

$$I_{\text{cr}} = \frac{1}{3}(12 \text{ in.})(6.52 \text{ in.})^3$$

$$+(7.04)(1.2 \text{ in}^2)(6.52 \text{ in.} - 2.5 \text{ in.})^2$$

$$+(8.04)(2.4 \text{ in}^2)(21.5 \text{ in.} - 6.52 \text{ in.})^2$$

$$= 5575.22 \text{ in}^4$$

Reinforced Concrete Sections - Example

The critical ratio of moment of inertia

$$\frac{I_{cr}}{I_g} = \frac{5575.22 \text{ in}^4}{16090.7 \text{ in}^4} = 0.346$$

$$I_{cr} \approx 0.35 I_g$$

Reinforced Concrete Sections - Example

Find the components of the beam

$$C_\mathrm{c} = 0.85 f_\mathrm{c} ba = 0.85(4\text{ ksi})(12\text{ in.})(0.85)c = 34.68c$$

$$\varepsilon'_\mathrm{s} = \left(\frac{c - 2.5\text{ in.}}{c}\right)(0.003) = 0.003 - \frac{0.0075}{c}$$

$$f_\mathrm{s} = E_\mathrm{s}\varepsilon'_\mathrm{s} = 29000\left(0.003 - \frac{0.0075}{c}\right) = 87 - \frac{217.5}{c}$$

$$C_\mathrm{s} = A'_\mathrm{s}\left(f_\mathrm{s} - 0.85 f_\mathrm{c}\right) = \left(1.2\text{ in}^2\right)\left(87 - \frac{217.5}{c}\right)$$

$$= 100.32 - \frac{261}{c}$$

Reinforced Concrete Sections - Example

Find the components of the beam

$$T = \left(2.4\ \text{in}^2\right)\left(60\ \text{ksi}\right) = 144\ \text{k}$$

$$T = C_\text{c} + C_\text{s}$$

$$144\ \text{k} = 34.68c + 100.32 - \frac{261}{c} \Rightarrow 34.68c^2 - 43.68c - 261 = 0$$

The neutral axis

$$c = \frac{43.68 + \sqrt{\left(43.68\right)^2 + 4\left(261\right)\left(34.68\right)}}{2\left(34.68\right)}$$

$$= 3.44\ \text{in.}$$

Reinforced Concrete Sections - Example

The strain of the steel

$$\varepsilon_s' = \left(\frac{3.44\text{ in.} - 2.5\text{ in.}}{3.44\text{ in.}}\right)(0.003) = 0.0008 \square \; 0.00207$$

$$\varepsilon_s = \left(\frac{21.5\text{ in.} - 3.44\text{ in.}}{3.44\text{ in.}}\right)(0.003) = 0.0158 \square \; 0.00207$$

<u>**Note**</u>: At service loads, beams are assumed to act elastically.

$$c = 3.44\text{ in.}$$

$$\bar{\text{y}} = 6.52\text{ in.}$$

Reinforced Concrete Sections - Example

Using a linearly varying ε and σ = Eε along the NA is the centroid of the area for an elastic center

$$\sigma = -\frac{My}{I}$$

The maximum tension stress in tension is

$$f_r = 7.5\sqrt{f_c} = 7.5\sqrt{4000}$$

$$= 474.3 \text{ psi} \quad \Rightarrow 0.4743 \text{ ksi}$$

Reinforced Concrete Sections - Example

The uncracked moments for the beam

$$\sigma = \frac{My}{I} \Rightarrow M = \frac{\sigma I}{y}$$

$$M_{\mathrm{cr}(+)} = \frac{f_{\mathrm{r}} I}{y} = \frac{0.4743\ \mathrm{ksi}\left(16090.7\ \mathrm{in}^4\right)}{\left(24\ \mathrm{in.} - 12.26\ \mathrm{in.}\right)} = 650.2\ \text{k-in.}$$

$$M_{\mathrm{cr}(-)} = \frac{f_{\mathrm{r}} I}{y} = \frac{0.4743\ \mathrm{ksi}\left(16090.7\ \mathrm{in}^4\right)}{12.26\ \mathrm{in.}} = 622.6\ \text{k-in.}$$

Calculate the Deflections

(1) Instantaneous (immediate) deflections

(2) Sustained load deflection

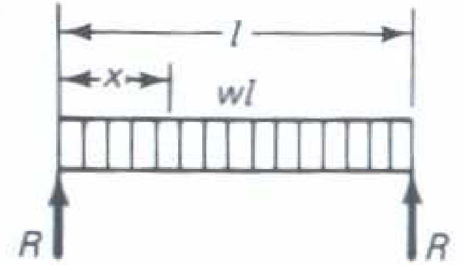
<u>Instantaneous Deflections</u>

due to dead loads(unfactored) , live, etc.

Calculate the Deflections

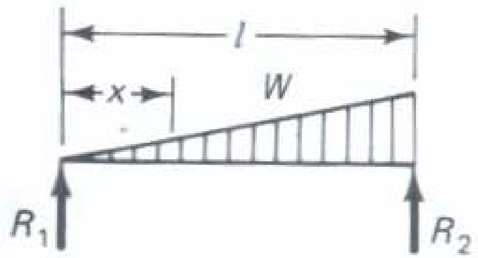
Instantaneous Deflections

Equations for calculating Δ_{inst} for common cases



$$M_x = \frac{wx}{2}(l - x)$$

$$\Delta_{max} \text{ (at center)} = \frac{5wl^4}{384EI}$$



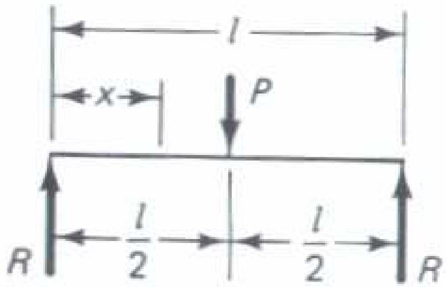
$$\Delta_{max} \left(\text{at } x = l\sqrt{1 - \sqrt{\tfrac{8}{15}}} = 0.5193l\right) = 0.01304\,\frac{Wl^3}{EI}$$

$$\Delta x = \frac{Wx}{180EI\,l^2}(3x^4 - 10l^2x^2 + 7l^4)$$

Calculate the Deflections

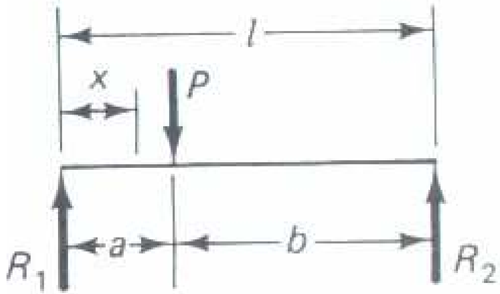
Instantaneous Deflections

Equations for calculating Δ_{inst} for common cases



$$\Delta_{max} \text{ (at point of load)} = \frac{Pl^3}{48EI}$$

$$\Delta x \left(\text{when } x < \frac{l}{2}\right) = \frac{Px}{48EI}(3l^2 - 4x^2)$$



$$\Delta_{max} \left(\text{at } x = \sqrt{\frac{a(a+2b)}{3}} \text{ when } a > b\right) = \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EI\,l}$$

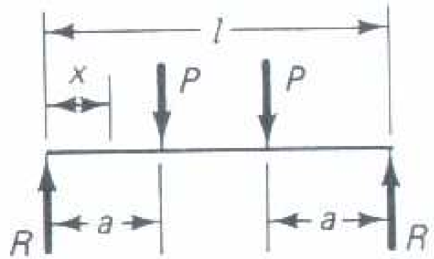
$$\Delta a \text{ (at point of load)} = \frac{Pa^2b^2}{3EI\,l}$$

$$\Delta x \text{ (when } x < a) = \frac{Pbx}{6EI\,l}(l^2 - b^2 - x^2)$$

Calculate the Deflections

Instantaneous Deflections

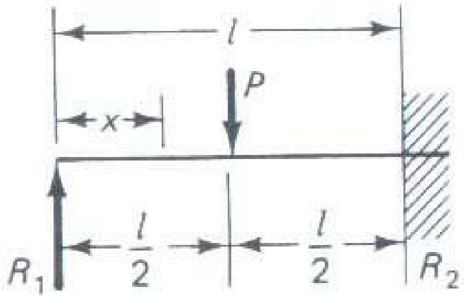
Equations for calculating Δ_{inst} for common cases



$$\Delta_{max} \text{ (at center)} = \frac{Pa}{24EI}(3l^2 - 4a^2)$$

$$\Delta x \text{ (when } x < a) = \frac{Px}{6EI}(3la - 3a^2 - x^2)$$

$$\Delta x \text{ (when } x > a \text{ and } < (l - a)) = \frac{Pa}{6EI}(3lx - 3x^2 - a^2)$$



$$\Delta_{max} \left(\text{at } x = l\sqrt{\tfrac{1}{5}} = 0.4472l\right) = \frac{Pl^3}{48EI\sqrt{5}} = 0.009317\frac{Pl^3}{EI}$$

$$\Delta x \text{ (at point of load)} = \frac{7Pl^3}{768EI}$$

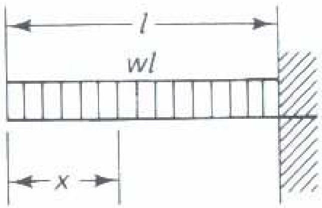
$$\Delta x \left(\text{when } x < \frac{l}{2}\right) = \frac{Px}{96EI}(3l^2 - 5x^2)$$

$$\Delta x \left(\text{when } x > \frac{l}{2}\right) = \frac{P}{96EI}(x - l)^2 (11x - 2l)$$

Calculate the Deflections

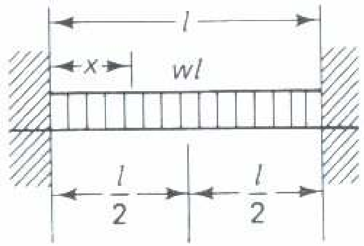
Instantaneous Deflections

Equations for calculating Δ_{inst} for common cases



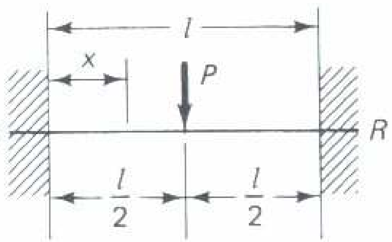
Δ_{max} (at free end) $= \dfrac{wl^4}{8EI}$

$\Delta x = \dfrac{w}{24EI}(x^4 - 4l^3x + 3l^4)$



Δ_{max} (at center) $= \dfrac{wl^4}{384EI}$

$\Delta x = \dfrac{wx^2}{24EI}(l - x)^2$



Δ_{max} (at center) $= \dfrac{Pl^3}{192EI}$

$\Delta x \left(\text{when } x < \dfrac{l}{2}\right) = \dfrac{Px^2}{48EI}(3l - 4x)$

Sustained Load Deflections

Creep causes an increase in concrete strain $\Rightarrow$ Curvature increases

Compression steel present → Increase in compressive strains cause increase in stress in compression reinforcement (reduces creep strain in concrete)

↓

Helps limit this effect.

Sustained Load Deflections

Sustain load deflection = $\lambda \, \Delta_i$

↑ Instantaneous deflection

$$\lambda = \frac{\xi}{1 + 50\rho'}$$

ACI 9.5.2.5

$$\rho' = \frac{A_s'}{bd}$$

at midspan for simple and continuous beams

at support for cantilever beams

Sustained Load Deflections

ξ = time dependent factor for sustained load

5 years or more	$\Rightarrow$ 2.0
12 months	$\Rightarrow$ 1.4
6 months	$\Rightarrow$ 1.2
3 months	$\Rightarrow$ 1.0

Also see Figure 9.5.2.5 from ACI code

Sustained Load Deflections

For dead and live loads

$$\Delta_{total} = \Delta_{DL(inst)} + \Delta_{LL(inst)} + \Delta_{DL(L.T.)} + \Delta_{LL(L.T.)}$$

DL and LL may have different ξ factors for LT (long term) Δ calculations

$$\Delta_{total\left(\begin{array}{c}\text{after attachment of}\\ \text{N/S components}\end{array}\right)} = \Delta_{total} - \Delta_{DL(inst)}$$

Sustained Load Deflections

The appropriate value of I_c must be used to calculate Δ at each load stage.

$\Delta_{DL(inst)} \longrightarrow$ Some percentage of DL (if given)

$\Delta_{LL(inst)} + \Delta_{DL(inst)} \longrightarrow$ Full DL and LL

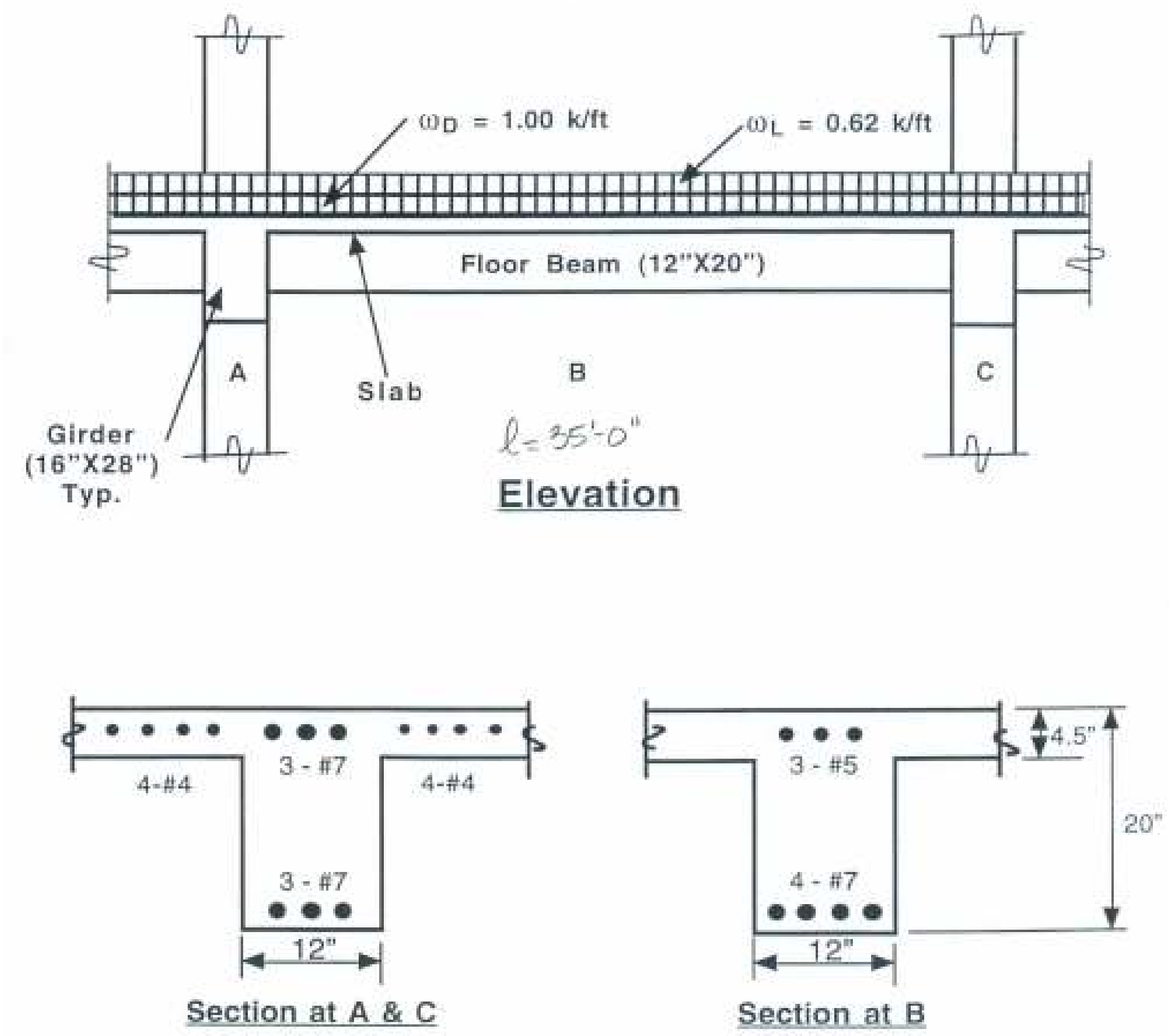
Serviceability Load Deflections - Example

Show in the attached figure is a typical interior span of a floor beam spanning between the girders at locations A and C. Partition walls, which may be damaged by large deflections, are to be erected at this level. The interior beam shown in the attached figure will support one of these partition walls. The weight of the wall is included in the uniform dead load provided in the figure. Assume that 15 % of the distributed dead load is due to a superimposed dead load, which is applied to the beam after the partition wall is in place. Also assume that 40 % of the live load will be sustained for at least 6 months.

Serviceability Load Deflections - Example

$f_c = 5$ ksi

$f_y = 60$ ksi



Serviceability Load Deflections - Example

Part I

Determine whether the floor beam meets the ACI Code maximum permissible deflection criteria. (Note: it will be assumed that it is acceptable to consider the effective moments of inertia at location A and B when computing the average effective moment of inertia for the span in this example.)

Part II

Check the ACI Code crack width provisions at midspan of the beam.

Serviceability Load Deflections - Example

Deflection before glass partition is installed (85 % of DL)

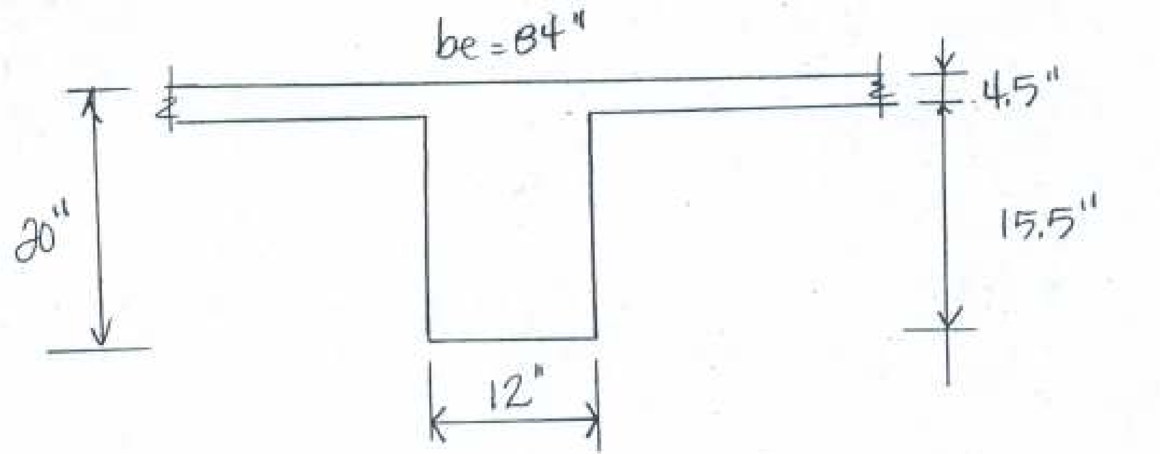
$$b_e \leq \frac{l}{4} = \frac{35\text{ft}\left(12\text{ in/ft}\right)}{4} = 105\text{ in}$$

$$\leq \left(8\text{t}\right)\left(2\right) + b_w = 8\left(4.5\text{ in}\right)2 + 12\text{ in} = 84\text{ in}$$

$$\leq \text{s} = 10\text{ ft} = 120\text{ in.}$$

Serviceability Load Deflections - Example

Compute the centroid and gross moment of inertia, I_g.



	b	h	Area	yi	Ai * yi
Flange	84	4.5	378	17.75	6709.5
Web	15.5	12	186	7.75	1441.5
			564		8151

Serviceability Load Deflections - Example

The moment of inertia

$$\bar{y} = \frac{\sum y_i A_i}{\sum A_i} = \frac{8151 \text{ in}^3}{564 \text{ in}^2} = 14.45 \text{ in} \cong 14.5 \text{ in}$$

$$I_{\text{g}} = \sum \left(\frac{1}{12} bh^3 + Ad^2 \right)$$

$$= \frac{1}{12}(84 \text{ in})(4.5 \text{ in})^3 + \left(378 \text{ in}^2\right)(17.75 \text{ in - } 14.45 \text{ in})^2$$

$$+ \frac{1}{12}(12 \text{ in})(15.5 \text{ in})^3 + \left(186 \text{ in}^2\right)(7.75 \text{ in - } 14.45 \text{ in})^2$$

$$= 16{,}950 \text{ in}^4 \cong 16900 \text{ in}^4$$

Serviceability Load Deflections - Example

The moment capacity

$$f_r = 7.5\sqrt{f_c} = 7.5\sqrt{5000} = 530 \text{ psi}$$

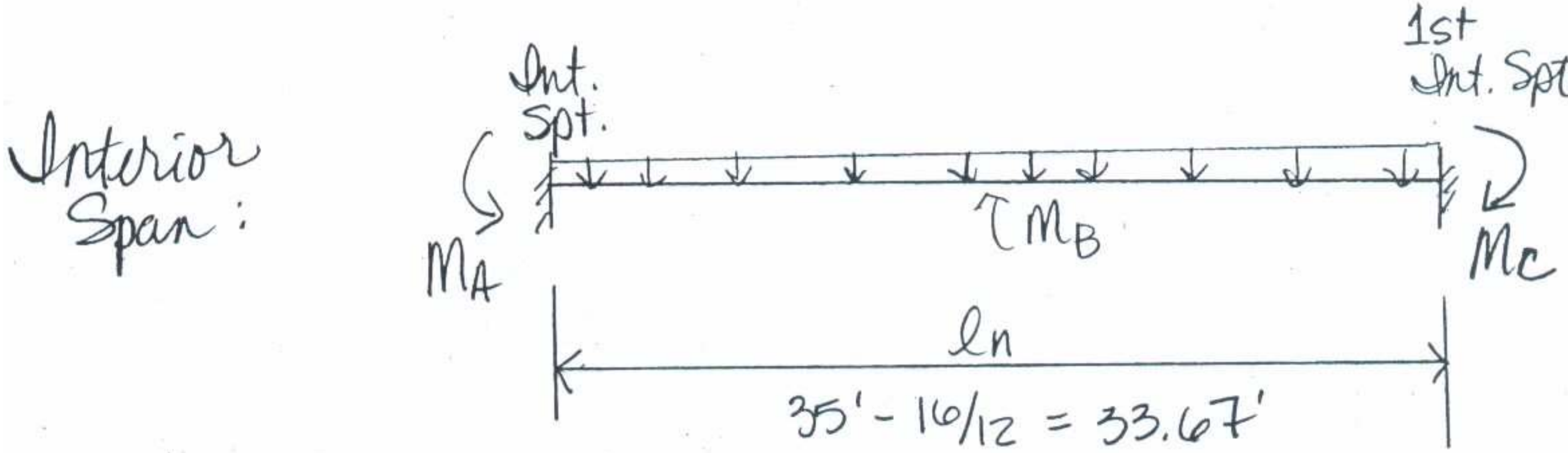
$$E_c = 57000\sqrt{f_c} = 57000\sqrt{5000} / 1000 \text{ lbs} = 4030 \text{ ksi}$$

$$M_{cr(-)} = \frac{f_r I_g}{y_{t(-)}} = \frac{(530 \text{ psi})(16900 \text{ in}^4)}{(5.55 \text{ in})(1000 \text{ lbs/kip})} = 1610 \text{ k-in}$$

$$M_{cr(+)} = \frac{f_r I_g}{y_{t(+)}} = \frac{(530 \text{ psi})(16900 \text{ in}^4)}{(14.5 \text{ in})(1000 \text{ lbs/kip})} = 618 \text{ k-in}$$

Serviceability Load Deflections - Example

Determine bending moments due to initial load (0.85 DL) The ACI moment coefficients will be used to calculate the bending moments Since the loading is not patterned in this case, This is slightly conservative



Serviceability Load Deflections - Example

The moments at the two locations

$$M_{\mathrm{A}} = -\left(0.85 w_{\mathrm{D}}\right)\left(l_{\mathrm{n}}\right)^2 / 11$$

$$= -0.85 * 1.00\text{k/ft} * \left(33.67 \text{ ft}\right)^2 / 11$$

$$= 87.6 \text{ k-ft} \Rightarrow 1050 \text{ k-in}$$

$$M_{\mathrm{B}} = \left(0.85 w_{\mathrm{D}}\right)\left(l_{\mathrm{n}}\right)^2 / 16$$

$$= -0.85 * 1.00\text{k/ft} * \left(33.67 \text{ ft}\right)^2 / 16$$

$$= 60.2 \text{ k-ft} \Rightarrow 723 \text{ k-in}$$

Serviceability Load Deflections - Example

Moment at C will be set equal to M_a for simplicity, as given in the problem statement.

$$M_A = 1050 \text{ k-in} < M_{cr(-)}$$

$$= 1610 \text{ k-in} \rightarrow \text{ Use } I_g \text{ @ supports}$$

$$M_B = 723 \text{ k-in} > M_{cr(+)}$$

$$= 618 \text{ k-in} \rightarrow \text{ Use } I_{cr} \text{ @ midspan}$$

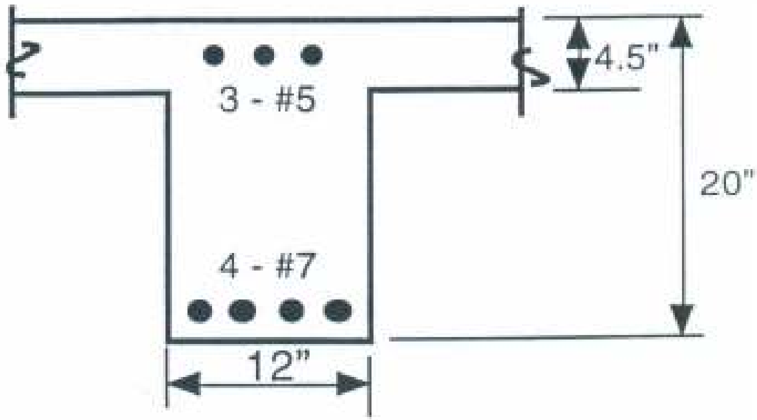
Serviceability Load Deflections - Example

Assume Rectangular Section Behavior and calculate the areas of steel and ratio of Modulus of Elasticity

$$n = \frac{E_s}{E_c} = \frac{29000 \text{ ksi}}{4030 \text{ ksi}} = 7.2$$

$$A_s' = 3\#5 = 3\left(0.31 \text{ in}^2\right) = 0.93 \text{ in}^2$$

$$A_s = 4\#7 = 4\left(0.6 \text{ in}^2\right) = 2.40 \text{ in}^2$$



Section at B

Serviceability Load Deflections - Example

Calculate the center of the T-beam

$$\bar{y}^2 + \left[\frac{2(n-1)A_s' + 2nA_s}{b} \right] \bar{y} - \left[\frac{2(n-1)A_s'd' + 2nA_s d}{b} \right] = 0$$

$$\bar{y}^2 + \left[\frac{2(6.2)\left(0.93 \text{ in}^2\right) + 2(7.2)\left(2.4 \text{ in}^2\right)}{84 \text{ in.}} \right] \bar{y}$$

$$-\left[\frac{2(6.2)\left(0.93 \text{ in}^2\right)(2.5 \text{ in.}) + 2(7.2)\left(2.4 \text{ in}^2\right)(17.5 \text{ in.})}{84 \text{ in.}} \right] = 0$$

$$\bar{y}^2 + 0.549\bar{y} - 7.54 = 0$$

$$\bar{y} = \frac{-0.549 \pm \sqrt{30.47}}{2} = 2.49 \text{ in.}$$

Serviceability Load Deflections - Example

The centroid is located at the $A'_s < 4.5$ in. $= t_f$ Use rectangular section behavior

$$I_{cr(+)} = \frac{1}{3} b\overline{y}^3 + (n-1) A'_s (\overline{y} - d')^2 + nA_s (d - \overline{y})^2$$

$$= \frac{1}{3}(84 \text{ in})(2.49 \text{ in})^3$$

$$+(6.2)(0.93 \text{ in}^2)(2.5 \text{ in} - 2.49 \text{ in})^2$$

$$+(7.2)(2.4 \text{ in}^2)(17.5 \text{ in} - 2.49 \text{ in})^2$$

$$= 4330 \text{ in}^4$$

Serviceability Load Deflections - Example

The moment of inertia at midspan

$$I_{e(\text{midspan})} = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left(1 - \left(\frac{M_{cr}}{M_a}\right)^3\right) I_{cr}$$

$$= \left(\frac{618 \text{ k-in}}{723 \text{ k-in}}\right)^3 \left(16900 \text{ in}^4\right)$$

$$+ \left(1 - \left(\frac{618 \text{ k-in}}{723 \text{ k-in}}\right)^3\right) \left(4330 \text{ in}^4\right)$$

$$= 12200 \text{ in}^4$$

Serviceability Load Deflections - Example

Calculate average effective moment of inertia, $I_{e(avg)}$ for interior span (for 0.85 DL) For beam with two ends continuous and use I_g for the two ends.

$$I_{e(avg)} = 0.7 I_{e(mid)} + 0.15\left(I_{e1} + I_{e2}\right)$$

$$= 0.7\left(12200 \text{ in}^4\right)$$

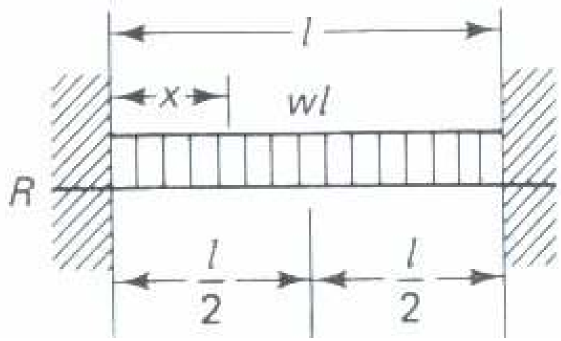
$$+0.15\left(16900 \text{ in}^4 + 16900 \text{ in}^4\right)$$

$$= 13600 \text{ in}^4$$

Serviceability Load Deflections - Example

<u>Calculate instantaneous deflection due to 0.85 DL:</u>

Use the deflection equation for a fixed-fixed beam but use the span length from the centerline support to centerline support to reasonably approximate the actual deflection.



Δ_{max} (at center) $= \dfrac{wl^4}{384EI}$

$\Delta x = \dfrac{wx^2}{24EI}(l-x)^2$

$$\Delta_{DL(inst)} = \frac{\omega l^4}{384EI} = \frac{0.85(1.00\text{ k/ft})(35\text{ ft})^4(12\text{ in/ft})^3}{384(4030\text{ ksi})(13600\text{ in}^4)}$$

$$= 0.105\text{ inches}$$

Serviceability Load Deflections - Example

Calculate additional short-term Deflections (full DL & LL)

$$M_A = \frac{-\left(\omega_D + \omega_L\right) l_n^2}{11} = \frac{-\left(1.62 \text{ k/ft}\right)\left(33.67 \text{ ft}\right)^2}{11}$$

$$= -167 \text{ k-ft} = -2000 \text{ k-in}$$

$$M_B = \frac{\left(\omega_D + \omega_L\right) l_n^2}{16} = \frac{\left(1.62 \text{ k/ft}\right)\left(33.67 \text{ ft}\right)^2}{16}$$

$$= 115 \text{ k-ft} = -1380 \text{ k-in}$$

Serviceability Load Deflections - Example

Calculate additional short-term Deflections (full DL & LL)

Let $M_c = M_a = -2000$ k-in for simplicity see problem statement

$$M_{\mathrm{c}} = M_{\mathrm{A}} = \left|-2000 \text{ k-in}\right| > M_{\mathrm{cr(-)}} = \left|-1610 \text{ k-in}\right|$$

$$\Rightarrow \text{cracking at supports}$$

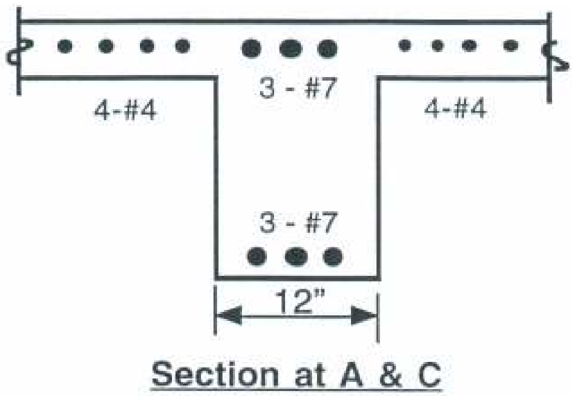
$$M_{\mathrm{B}} = 1380 \text{ k-in} > M_{\mathrm{cr(+)}} = 618 \text{ k-in}$$

$$\Rightarrow \text{cracking at midspan}$$

Serviceability Load Deflections - Example

Assume beam is fully cracked under full DL + LL, therefore I= I_{cr} (do not calculate I_e for now).

I_{cr} for supports



$$n = 7.20$$

$$A_s = 2(4)\left(0.20\text{ in}^2\right) + 3\left(0.6\text{ in}^2\right)$$

$$= 3.40\text{ in}^2$$

$$d = 20\text{ in} - 2.5\text{ in} = 17.5\text{ in}$$

$$A_s' = 3\left(0.6\text{ in}^2\right) = 1.80\text{ in}^2$$

$$d' = 2.5\text{ in}$$

Serviceability Load Deflections - Example

Class formula using doubly reinforced rectangular section behavior.

$$\bar{y}^2 + \left[\frac{2(n-1)A'_s + 2nA_s}{b} \right] \bar{y} - \left[\frac{2(n-1)A'_s d' + 2nA_s d}{b} \right] = 0$$

$$\bar{y}^2 + \left[\frac{2(6.2)(1.80 \text{ in}^2) + 2(7.2)(3.4 \text{ in}^2)}{12 \text{ in}} \right] \bar{y}$$

$$- \left[\frac{2(6.2)(1.80 \text{ in}^2)(2.5 \text{ in}) + 2(7.2)(3.4 \text{ in}^2)(17.5 \text{ in})}{12 \text{ in}} \right] = 0$$

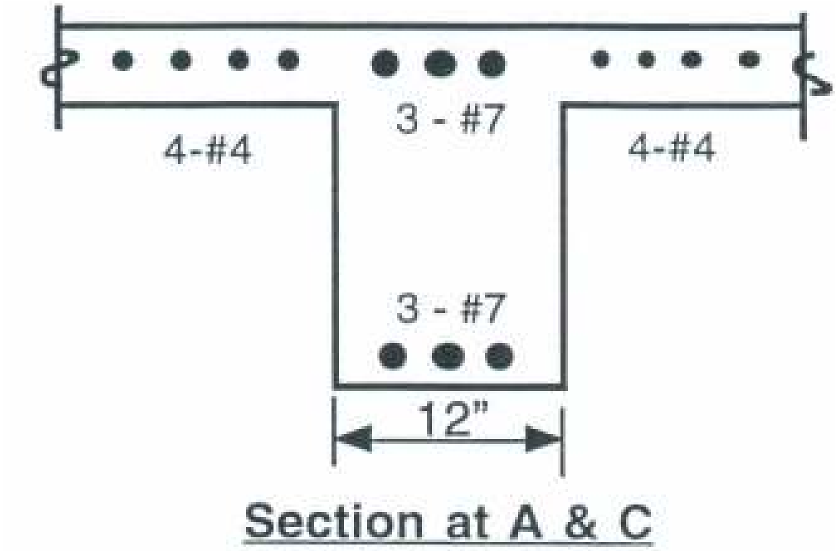
Serviceability Load Deflections - Example

Class formula using doubly reinforced rectangular section behavior.

$$\bar{y}^2 + \left[\frac{2(n-1)A'_s + 2nA_s}{b}\right]\bar{y} - \left[\frac{2(n-1)A'_s d' + 2nA_s d}{b}\right] = 0$$

$$\bar{y}^2 + 5.94\bar{y} - 76.05 = 0$$

$$\bar{y} = \frac{-5.94 \pm \sqrt{340}}{2} = 6.25 \text{ in.}$$



Serviceability Load Deflections - Example

Calculate moment of inertia.

$$I_{\mathrm{cr}(+)} = \frac{1}{3} b\overline{y}^{3} + (n-1) A'_{\mathrm{s}} \left(\overline{y} - d'\right)^{2} + nA_{\mathrm{s}} \left(d - \overline{y}\right)^{2}$$

$$= \frac{1}{3}(12 \text{ in})(6.25 \text{ in})^{3}$$

$$+(6.2)\left(1.80 \text{ in}^{2}\right)(2.5 \text{ in} - 6.25 \text{ in})^{2}$$

$$+(7.2)\left(3.4 \text{ in}^{2}\right)(17.5 \text{ in} - 6.25 \text{ in})^{2}$$

$$= 4230 \text{ in}^{4}$$

Serviceability Load Deflections - Example

Weighted I_{cr}

$$I_{\text{cr}} = 0.7 I_{\text{cr(mid)}} + 0.15\left(I_{\text{cr1}} + I_{\text{cr2}}\right)$$

$$= 0.7\left(4330 \text{ in}^4\right) + 0.15\left(4230 \text{ in}^4 + 4230 \text{ in}^4\right)$$

$$= 4300 \text{ in}^4$$

Serviceability Load Deflections - Example

Instantaneous Dead and Live Load Deflection.

$$\Delta_{\mathrm{DL(inst)}} = \frac{\omega_{\mathrm{D}} l^4}{384EI} = \frac{\left(1.00\ \mathrm{k/ft}\right)\left(35\ \mathrm{ft}\right)^4\left(12\ \mathrm{in/ft}\right)^3}{384\left(4030\ \mathrm{ksi}\right)\left(4300\ \mathrm{in}^4\right)}$$

$$= 0.390\ \mathrm{inches}$$

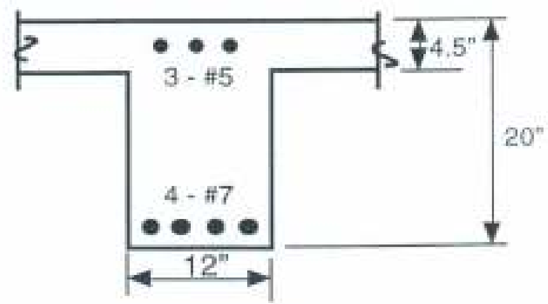
$$\Delta_{\mathrm{LL(inst)}} = \Delta_{\mathrm{DL(inst)}} * \frac{0.62\ \mathrm{k/ft}}{1.00\ \mathrm{k/ft}}$$

$$= 0.242\ \mathrm{inches}$$

Serviceability Load Deflections - Example

Long term Deflection at the midspan

$$\rho'(t) = \frac{A_s(t)}{2b_w d} = \frac{3\left(0.31\ \text{in}^2\right)}{2(12\ \text{in})(17.5\ \text{in})} = 0.00221$$



Section at B

Dead Load (Duration > 5 years)

$$\lambda = \frac{\xi}{1+50\rho'} = \frac{2.0}{1+50(0.00221)} = 1.80$$

$$\Delta_{\text{DL(L.T.)}} = \lambda_{\text{DL}}\Delta_{\text{DL(inst)}} = 1.8(0.390\ \text{in}) = 0.702\ \text{in}$$

Serviceability Load Deflections - Example

Long term Deflection use the midspan information

$$\rho'(t) = \frac{A_s(t)}{2b_w d} = \frac{3(0.31\text{ in}^2)}{2(12\text{ in})(17.5\text{ in})} = 0.00221$$

Live Load (40 % sustained 6 months)

$$\lambda = \frac{\xi}{1+50\rho'} = \frac{1.2}{1+50(0.00221)} = 1.08$$

$$\Delta_{\text{LL(L.T.)}} = \lambda_{\text{LL}}\Delta_{\text{LL(inst)}} = 1.08(0.242\text{ in})(0.40)$$

$$= 0.105\text{ in}$$

Serviceability Load Deflections - Example

Total Deflection after Installation of Glass Partition Wall.

$$\begin{aligned}\Delta_{\text{total}} &= \Delta_{\text{DL(inst)}} + \Delta_{\text{LL(inst)}} + \Delta_{\text{DL(L.T.)}} + \Delta_{\text{LL(L.T.)}} \\ &= 0.390 \text{ in} + 0.242 \text{ in} + 0.702 \text{ in} + 0.105 \text{ in} \\ &= 1.44 \text{ in}\end{aligned}$$

$$\begin{aligned}\Delta_{\text{after attachment}} &= \Delta_{\text{total}} - \Delta_{\text{DL(inst)}} \\ &= 1.44 \text{ in} - 0.105 \text{ in} = 1.33 \text{ in}\end{aligned}$$

$$\begin{aligned}\Delta_{\text{permissible}} &= \frac{l}{480} \\ &= \frac{35 \text{ ft}\left(12 \text{ in/ft}\right)}{480} = 0.875 \text{ in} < 1.33 \text{ in} \;\left(\text{NO GOOD!}\right)\end{aligned}$$

Serviceability Load Deflections - Example

Check whether modifying I_{cr} to I_e will give an acceptable deflection:

$$I_{e(\text{midspan})} = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left(1 - \left(\frac{M_{cr}}{M_a}\right)^3\right) I_{cr}$$

$$= \left(\frac{618 \text{ k-in}}{1380 \text{ k-in}}\right)^3 \left(16900 \text{ in}^4\right)$$

$$+ \left(1 - \left(\frac{618 \text{ k-in}}{1380 \text{ k-in}}\right)^3\right)\left(4330 \text{ in}^4\right)$$

$$= 5460 \text{ in}^4$$

Serviceability Load Deflections - Example

Check whether modifying I_{cr} to I_e will give an acceptable deflection:

$$I_{e(support)} = \left(\frac{1610 \text{ k-in}}{2000 \text{ k-in}}\right)^3 \left(16900 \text{ in}^4\right)$$

$$+\left(1-\left(\frac{1610 \text{ k-in}}{2000 \text{ k-in}}\right)^3\right)\left(4230 \text{ in}^4\right)$$

$$= 10800 \text{ in}^4$$

$$I_{e(avg)} = 0.7 I_{e(mid)} + 0.15\left(I_{e1} + I_{e2}\right)$$

$$= 0.7\left(5460 \text{ in}^4\right) + 0.15\left(10800 \text{ in}^4 + 10800 \text{ in}^4\right)$$

$$= 7060 \text{ in}^4$$

Serviceability Load Deflections - Example

Floor Beam meets the ACI Code Maximum permissible Deflection Criteria. Adjust deflections:

$$\Delta_{\mathrm{DL(inst)}} = 0.390\ \mathrm{in}\left(\frac{4300\ \mathrm{in}^4}{7060\ \mathrm{in}^4}\right) = 0.238\ \mathrm{in}$$

$$\Delta_{\mathrm{LL(inst)}} = 0.242\ \mathrm{in}\left(\frac{4300\ \mathrm{in}^4}{7060\ \mathrm{in}^4}\right) = 0.147\ \mathrm{in}$$

$$\Delta_{\mathrm{DL(L.T.)}} = 0.702\ \mathrm{in}\left(\frac{4300\ \mathrm{in}^4}{7060\ \mathrm{in}^4}\right) = 0.428\ \mathrm{in}$$

$$\Delta_{\mathrm{LL(L.T.)}} = 0.105\ \mathrm{in}\left(\frac{4300\ \mathrm{in}^4}{7060\ \mathrm{in}^4}\right) = 0.064\ \mathrm{in}$$

Serviceability Load Deflections - Example

Adjust deflections:

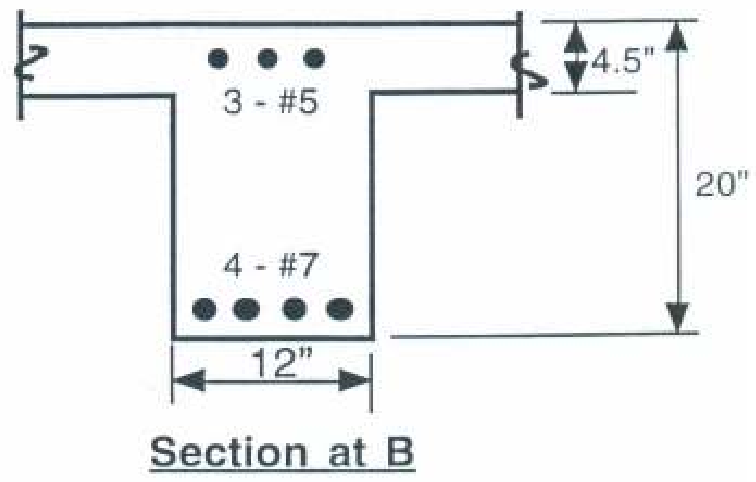
$$\begin{aligned}\Delta_{\text{total}} &= \Delta_{\text{DL(inst)}} + \Delta_{\text{LL(inst)}} + \Delta_{\text{DL(L.T.)}} + \Delta_{\text{LL(L.T.)}} \\ &= 0.238 \text{ in.} + 0.147 \text{ in.} + 0.428 \text{ in.} + 0.064 \text{ in.} \\ &= 0.877 \text{ in.}\end{aligned}$$

$$\begin{aligned}\Delta_{\text{after attachment}} &= \Delta_{\text{total}} - \Delta_{\text{DL(inst)}} \\ &= 0.877 \text{ in.} - 0.105 \text{ in.} \\ &= 0.772 \text{ in.} < \Delta_{\text{permissible}} = 0.875 \text{ in.} \quad (\text{OKAY!})\end{aligned}$$

Serviceability Load Deflections - Example

Part II: Check crack width @ midspan

$$z = f_s \sqrt[3]{d_c A}$$



$$d_c = d_s = 2.5 \text{ in}$$

$$A_e = 2d_s b = 2(2.5 \text{ in})(12 \text{ in}) = 60 \text{ in}^2$$

$$A = \frac{A_e}{\# \text{ bars}} = \frac{60 \text{ in}^2}{4} = 15 \text{ in}^2$$

Serviceability Load Deflections - Example

Assume

$$f_{\mathrm{s}} = 0.6 f_{\mathrm{y}} = 0.6\left(60\ \text{ksi}\right) = 36\ \text{ksi} \quad \left(\text{ACI } 10.6.4\right)$$

$$z = \left(36\ \text{ksi}\right)\sqrt[3]{\left(2.5\ \text{in}\right)\left(15\ \text{in}^2\right)}$$

$$= 120\ \text{k/in} < 175\ \text{k/in}\ \left(\text{OK}\right)$$

For interior exposure, the crack width @ midspan is acceptable.